



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

SEPTEMBER 2019

MATHEMATICS P2

MARKS: 150

TIME: 3 hours



* M A T H E 2 *

This question paper consists of 14 pages, including 1 page information sheet, and
an answer book of 25 pages.

INSTRUCTIONS AND INFORMATION


Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The table below shows the height, in metres, of 80 giraffes.

Height in m	Frequency
$4,6 \leq h < 4,8$	4
$4,8 \leq h < 5,0$	7
$5,0 \leq h < 5,2$	15
$5,2 \leq h < 5,4$	33
$5,4 \leq h < 5,6$	17
$5,6 \leq h < 5,8$	4



[Image from *Stuff You Should Know*]

- 1.1 Calculate the estimated mean height of the giraffes. (2)
- 1.2 The height range of $4,8 \leq h < 5,0$ m, consisted entirely of 7 young male giraffes. The mean of the heights of the 7 young males was calculated to be 4,86 m. However, it was noticed that the height of one of the young males was incorrectly recorded as 4,98 m but should have been 4,89 m. Calculate the new mean of the 7 young males. (3)
- 1.3 Would you expect the standard deviation of the individual heights of the 80 giraffes to be more or less than the standard deviation of the individual heights of the 7 young male giraffes? Explain your answer. (2)

[7]

QUESTION 2

Cocoa solids are used to make chocolate.

A student is investigating the relationship between the percentage of cocoa solids in a 100 g slab of chocolate and the price of the slab (in rands). The data obtained is shown in the table below.



Chocolate brand	A	B	C	D	E	F	G	H
x (% of cocoa)	10	20	30	35	40	50	60	70
y (price in rand)	6,50	10,20	9,40	24,00	11,20	16,80	20,50	24,20

- 2.1 Use the grid in the ANSWER BOOK to represent this data in a scatter plot. (2)
- 2.2 Determine the equation of the least squares regression line for this data. (3)
- 2.3 Draw the least squares regression line in the ANSWER BOOK on the same grid used in QUESTION 2.1. (2)
- 2.4 Calculate the correlation coefficient for the percentage of cocoa solids and the price of the chocolate slab. (1)
- 2.5 Comment on the relationship between the percentage of cocoa solids and the price of the chocolate slab. (1)
- 2.6 The student believes one brand of chocolate is overpriced.
- 2.6.1 Identify the chocolate brand which is over-priced. (1)
- 2.6.2 Estimate by how much more this slab of chocolate is overpriced. (3)

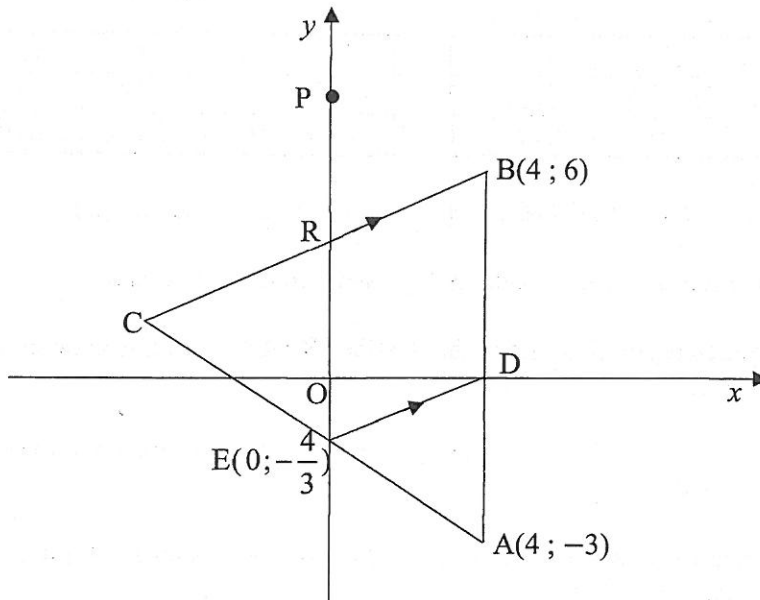
[13]

QUESTION 3

In the diagram below, $\triangle ABC$ is shown with coordinates of $A(4; -3)$ and $B(4; 6)$ given. AB intersects the x -axis at D and AC intersects the y -axis at E .

The coordinates of E are $(0; -\frac{4}{3})$. BC intersects the y -axis at R . P is a point on the y -axis.

$DE \parallel BC$.



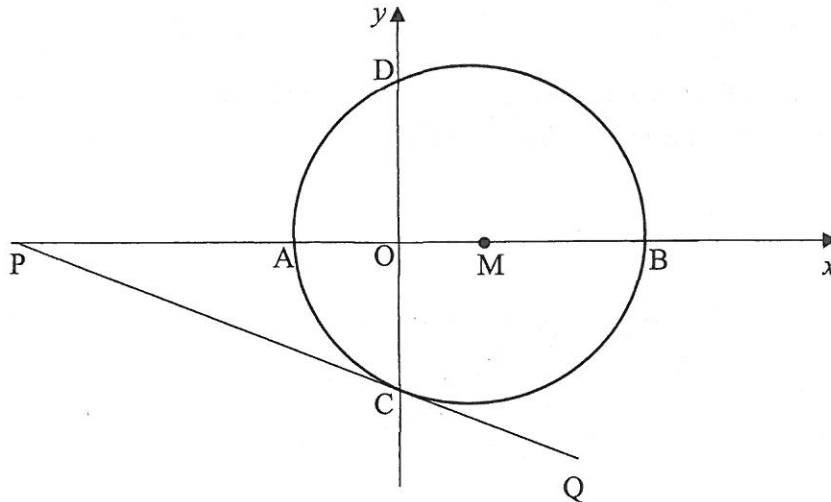
- 3.1 Write down the coordinates of D . (1)
- 3.2 Determine the gradient of DE . (2)
- 3.3 Calculate \widehat{PRB} . (4)
- 3.4 Calculate the length of DE in simplest surd form. (2)
- 3.5 Determine the ratio $AD: AB$ in its simplest form. (2)
- 3.6 Hence, or otherwise, calculate the length of BC . (3)
- 3.7 Determine:
 - 3.7.1 The midpoint of DE (2)
 - 3.7.2 The equation of the perpendicular bisector of DE in the form $y = mx + c$ (3)
- 3.8 Does this perpendicular bisector pass through A ? Justify your answer. (2)

[21]

QUESTION 4

The diagram below shows a circle having centre M which intersects the x -axis at A and B and the y -axis at D and C. PCQ is a tangent to the circle at C, the point of contact on the y -axis. P lies on the x -axis.

The equation of the circle is: $x^2 + y^2 - 6x - 16 = 0$



- 4.1 Show that the coordinates of M are (3; 0) and that the radius is 5 units. (3)
- 4.2 Determine:
- 4.2.1 The coordinates of B (3)
- 4.2.2 The coordinates of C (2)
- 4.3 If the length of PM is $8\frac{1}{3}$ units, calculate the length of PC. (3)
- 4.4 Calculate the angle subtended by the chord DC at B, i.e. find \widehat{DBC} . (4)
- 4.5 If the given circle is moved 2 units right and 1 unit up, determine the equation of the tangent to the circle in its new position passing through point C'. (4)

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QUESTION 5

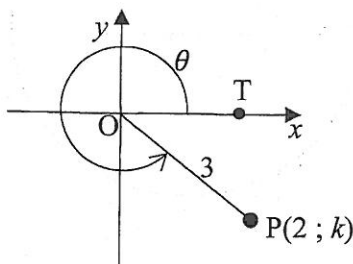
DO NOT USE A CALCULATOR FOR THIS QUESTION.

5.1 Complete the following identities:

5.1.1 $\cos^2 A + \sin^2 A = \dots$ (1)

5.1.2 $\cos^2 A - \sin^2 A = \dots$ (1)

5.2 P(2 ; k) is a point in the Cartesian plane such that OP = 3 units and reflex angle $T\hat{O}P = \theta$, as shown in the diagram below.



5.2.1 Calculate the value of k (leave your answer in surd form). (2)

5.2.2 Hence, determine the value of the following:

(a) $\tan(\theta - 180^\circ)$ (2)

(b) $\frac{1 - \sin^2 2\theta}{1 - 2\sin^2 \theta}$ (4)

5.3 Determine, **without the use of a calculator**, the value of:

$\sin(-200^\circ) \cdot \cos 310^\circ + \tan(-135^\circ) \cdot \cos 380^\circ \cdot \sin 230^\circ$ (6)

5.4 Prove the following identity:

$\sin 2\theta + \cos(2\theta - 90^\circ) = 4 \sin \theta \cos \theta$ (3)

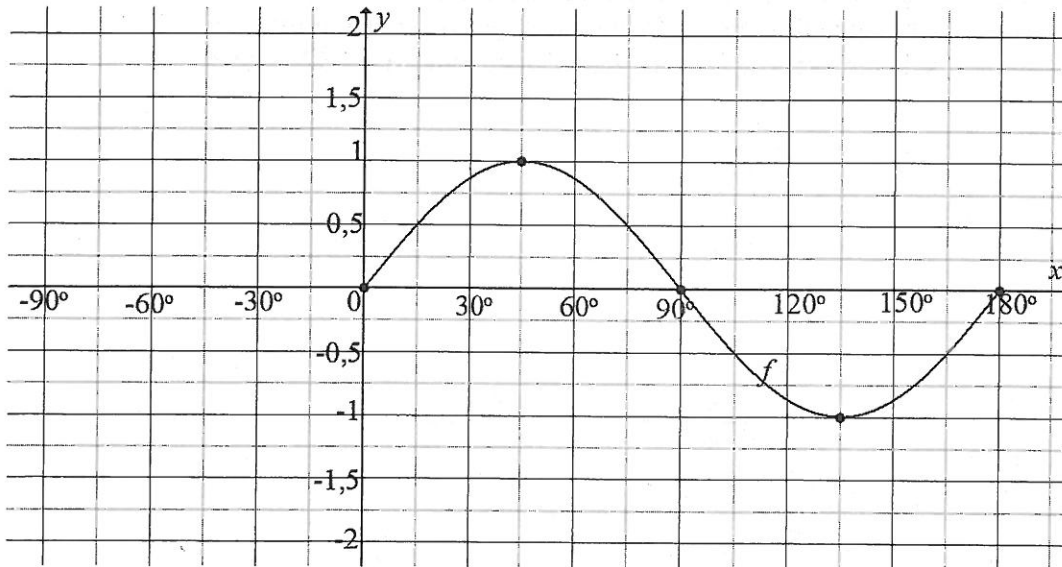
5.5 Solve for x if:

$10^{\sin x} + 10^{\sin x + 1} = 110$ for $-360^\circ \leq x \leq 360^\circ$ (5)

[24]

QUESTION 6

The graph below shows part of the function $f(x) = \sin 2x$ for $0^\circ \leq x \leq 180^\circ$.



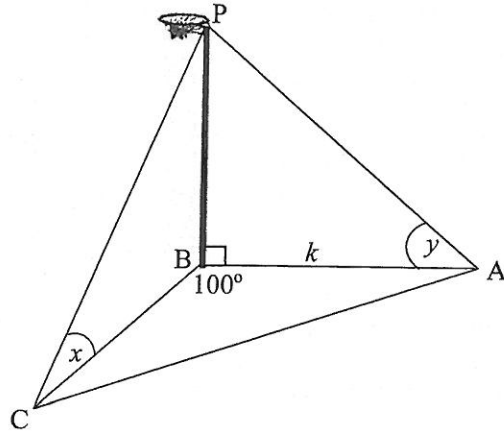
- 6.1 On the grid provided in the ANSWER BOOK, complete the graph of f for the interval $-90^\circ \leq x \leq 180^\circ$. (1)
- 6.2 On the same grid, draw the graph of $g(x) = \cos(x - 30^\circ)$ for the interval $-90^\circ \leq x \leq 180^\circ$. Clearly show the intercepts with the axes, the coordinates of the turning points and the end points of the graph. (4)
- 6.3 Calculate the solutions to the equation:

$$\sin 2x = \cos(x - 30^\circ) \quad \text{for } -90^\circ \leq x \leq 90^\circ \quad (6)$$

[11]

QUESTION 7

The diagram below shows a vertical netball pole PB. Player A is standing on the base line of the court and the angle of elevation from A to the top of the pole P is y° . A second player is standing in the court at C, and the angle of elevation from C to P is x° . Points A, B and C are in the same horizontal plane. BA is k metres; $\widehat{ABC} = 100^\circ$.

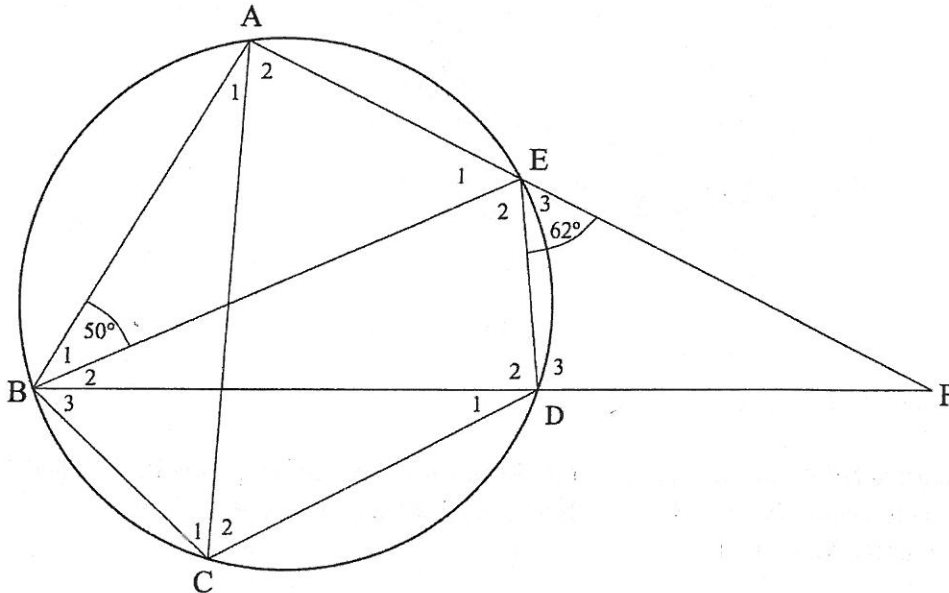


- 7.1 Show that $BC = \frac{k \cdot \tan y}{\tan x}$ (3)
- 7.2 Calculate the length of AC if $BC = 4,73$ m and $k = 3$ m. (3)
- [6]**

Give reasons for your statements in QUESTIONS 8, 9, and 10.

QUESTION 8

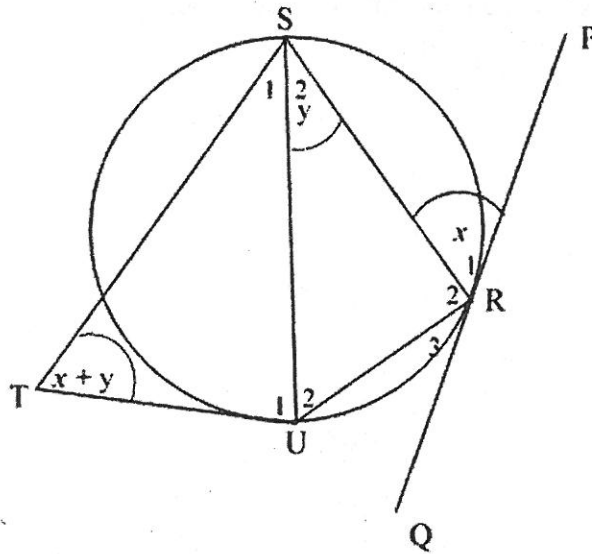
- 8.1 In the given diagram, A, B, C, D and E are points on the circle.
 BE is a diameter. $\widehat{E}_3 = 62^\circ$ and $\widehat{B}_1 = 50^\circ$.
 BD produced meets AE produced at F.



Determine, with reasons:

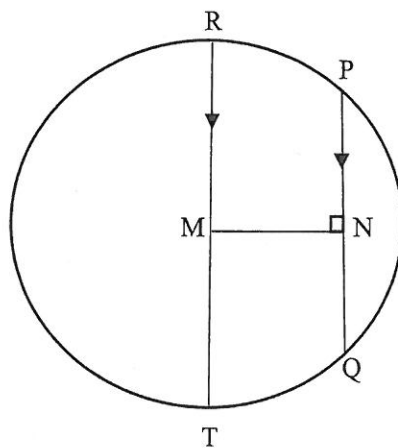
- 8.1.1 \widehat{BAE} (2)
- 8.1.2 \widehat{E}_1 (2)
- 8.1.3 \widehat{C}_1 (2)
- 8.1.4 \widehat{C}_2 (2)
- 8.1.5 \widehat{ABD} (2)
- 8.2 Complete the following theorem statement:
- The angle between the tangent to a circle and the chord drawn from the point of contact is ... (1)

- 8.3 In the diagram below, PRQ is a tangent to the circle SUR at R. SU, SR and UR are drawn. Lines from S and U produced meet at T outside the circle.
 $\widehat{R}_1 = x$; $\widehat{S}_2 = y$ and $\widehat{STU} = x + y$



Prove that STUR is a cyclic quadrilateral. (5)

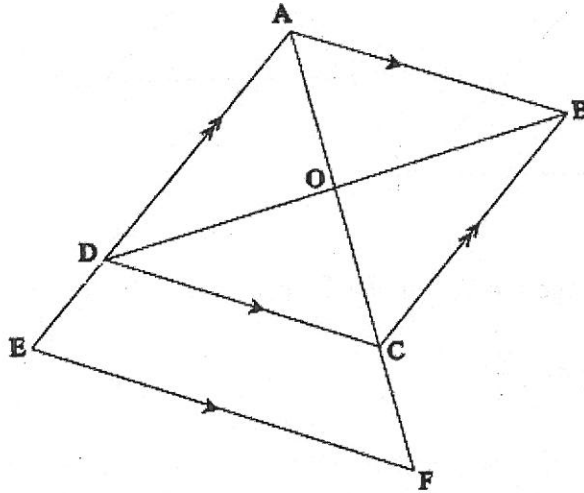
- 8.4 The diagram below shows a circle centre M passing through the points R, P, Q and T. RT is the diameter. PQ is a chord such that $PQ \parallel RT$ and $MN \perp PQ$. $PQ = 16$ units, $MN = 6$ units.



Determine the length of RT. (4)
[20]

QUESTION 9

In the diagram below, ABCD is a parallelogram. AD and AC are produced to E and F respectively so that $EF \parallel DC$. AF and DB intersect at O.
 AD = 12 units; DE = 3 units; DC = 14 units; CF = 5 units.



9.1 Calculate, giving reasons, the length of:

9.1.1 AC (3)

9.1.2 AO (1)

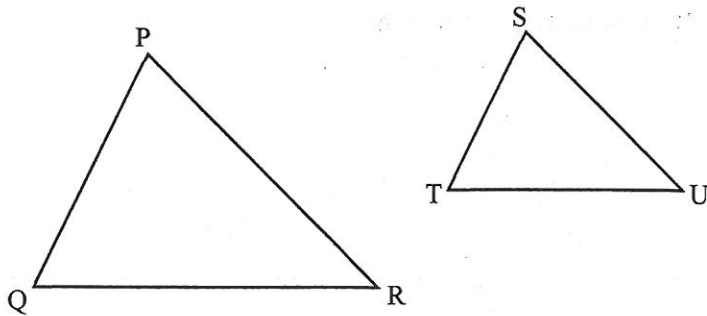
9.1.3 EF (3)

9.2 Prove that $\frac{\text{area } \triangle ADO}{\text{area } \triangle AEF} = \frac{8}{25}$ (3)

[10]

QUESTION 10

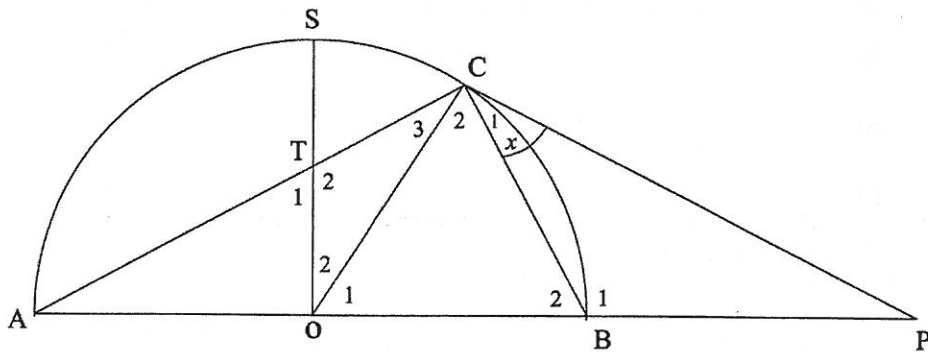
10.1 For the triangles shown below,



prove that if $\widehat{P} = \widehat{S}$ and $\widehat{Q} = \widehat{T}$ and $\widehat{R} = \widehat{U}$ then

$$\frac{ST}{PQ} = \frac{SU}{PR} \quad (6)$$

10.2 In the diagram below, O is the centre of a semi-circle ACB. S is a point on the circumference and T lies on AC such that $STO \perp AB$. Diameter AB is produced to P, such that PC is a tangent to the semi-circle at C. Let $\widehat{C}_1 = x$.



10.2.1 Write down, with reasons, 2 other angles equal to x . (3)

10.2.2 Prove that $\triangle TOC \parallel \triangle BPC$ (5)

10.2.3 Prove that $TO \cdot PC = OB \cdot BP$ (2)

10.2.4 If $BP = OB$, show that $3OC^2 = PC^2$ (3)

[19]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\begin{aligned} \text{In } \Delta ABC: \quad \frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cdot \cos A \\ \text{area } \Delta ABC &= \frac{1}{2} ab \cdot \sin C \end{aligned}$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$